THE MATHEMATICAL OPERATION OF URINE FLOW RATE, PRESSURE, URETHRAL RESISTANCE, URETHRAL DIAMETER AND URETHRAL LENGTH IN MICTURITION

Hypothesis / aims of study
The aim of this study was to test and verify the mathematical relationship among urine flow rate, pressure, urethral resistance, urethral diameter and urethral length in micturition. These parameters were discovered as mathematical logic for qualitative, quantitative and orientational relationship according to Blasius’s equation[1], which is at Qmax time:

\[ P_{dif} / Q^2 = R = 258\pi L / [(5000Q / F)^{0.25} - F] \]

The significance of the pressure difference (Pdif) is intravesical pressure (Pves) minus urethral outlet pressure (Po). The urethral resistance (R) means the physical quantity of urethra to resist urine flow. The F originate from the diameter of urethra (F=20\pi r=10md) at the narrowest channel of urethra. The L is urethral length and the Q is urine flow rate.

Study design, materials and methods
Between February 2008 and January 2011, a total of 93 males distributed in 5 groups retrospectively, at a mean age of 58.4 years (range 38 to 76), were performed urodynamic and video-urodynamic evaluation to assess the mathematical relationship among those parameters. 16 males in group A had normal micturition without any LUTS. 23 patients in group B with BPH and BOO were detected high Pdet and low flow rate for micturition. 36 patients who had undergone TUR-P because of BPH formed group C. 6 patients with diagnosis of anterior urethral stricture formed group D. 12 patients in group E with BPH III and BOO were detected low Pdet and low flow rate. All 93 cases were performed urodynamic and video-urodynamic tests to measure the pressure (Pves, Pabd, Pdet, Po, Pdif), the urethral diameter and the urethral length.

The methods we had done the tests were variant in group A and the others. In group A, we measured Pves by 8Fr catheter via urethra because of their illness did not need the condition of suprapubic cystotomy. But in group B, C, D and E, we measured Pves via suprapubic vesical fistula by an 8Fr balloon catheter. All the object’s bladder is filled with 300~400ml liquid of 12.5%NaI disinfectant solution at velocity around 30 ml/min. The Pabd was measured in rectal ampulla by a 12Fr balloon catheter via anus all the same. The urethral outlet pressure (Po) might be measured by an 8Fr catheter with a hole at the end of the tube, which maintains a distance about 0.5~1mm out of the urethral meatus. The X-ray cystourethrography images were synchronistically recorded on videotape machine. And there was a metal ruler placed at the inferior of the patient’s abdomen which marked 1Fr~15Fr and 0.2~10cm as a reference object to measure the urethral diameter and urethral length in cystourethrography.

After measuring, we used computer programs to verify whether the parameters can make equation established at Qmax time.

Results
11 of 16 cases’ parameters resulted from group A could make equality be founded (p>0.10). Other 5 cases could not (p<0.10). All parameters resulted from group B, C, D and E, at total of 77 cases, could make equality be founded (p>0.10).

Interpretation of results
The way we performed the test in group A might lead to the failure of those 5 cases. Firstly, when we measured Pves by 8Fr catheter via urethra into bladder and Po by catheter out of the urethra meatus synchronistically, catheterization might bring about Po lose. Secondly, the catheter of 8Fr in the urethra would take a certain space, hence in the video-urodynamic the cases in group A might get mistakes which could overestimate the urethral diameter. By contrast, cases in group B, C, D and E who measured Pves by 8Fr catheter via suprapubic vesical fistula did not meet those problems. So all their parameters from urodynamic and video-urodynamic tests were conformed to the mathematical operation of Blasius’s equation[1].

Concluding message
Based on our data, our experience indicates that the urethral resistance (R) could be measured in two different way: the combination of pressure difference (Pdif) and squared flow rate (Q^2) from pressure flow studies (Pdif / Q^2) or calculated by the data of urethral diameter, length and flow rate base on cystourethrography at Qmax time. The unit of the urethral resistance (R) could be expressed as cmH2O·ml^{-2}·sec^{-2} or cm^{-2}·sec and it is abbreviated as H (H = cmH2O·ml^{-2}·sec^{-2} = cm^{-2}·sec^{-2}). We advice the critical quantitative value of R of obstruction at 0.5H, which means BOO may be defined if R was greater than 0.5H at Qmax time. However, the validity of this mathematical relationship still needs more investigations with larger number of patients and variety of voiding dysfunction. From these results, cystourethrography and flow rate study may diagnose BOO with its noninvasive character which could not be obtained by other examinations.

The diagrammatic drawing of each parameter (Q, R, a0, ±Pa, Pves, Pdet, Pabd, Pdif, Pw, Po, F) is displayed below (Fig 1) in time of micturition. Detailed formula derivation will be shown in the full text.
Fig 1. “Pdet + Pabd = Pves” is bladder pressure which forms the source of the pressure. “Pw ± Pa + Po = Pdif + Po” is the changed form of bladder pressure. At Qmax time (Pa = 0), Blasius’s equation[1] can be expressed as
\[ P_{dif} = \frac{258\pi L \cdot Q^2}{\left(\frac{5000Q}{F}\right)^{0.25} \cdot F^5}, \]
which generate
\[ R = \frac{P_{dif}}{Q^2} = \frac{258\pi L}{\left(\frac{5000Q}{F}\right)^{0.25} \cdot F^5}. \]

References

Disclosures
Funding: NONE  Clinical Trial: No  Subjects: HUMAN  Ethics not Req’d: It is a retrospective study  Helsinki: Yes  Informed Consent: Yes