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CALCULATION OF BLADDER WALL THICKNESS CHANGES IN A SYMMETRICALLY EXPANDING, NON-SPHERICAL BLADDER

Aims of Study: The size of the bladder wall is an important factor in the calculation of bladder wall tension. When the bladder fills, common-sense knowledge is that the wall thickness is reducing. The computation of this reduction is less straightforward and most authors use the model of a symmetrically expanding spherical bladder to approximate the real behaviour of the expanding bladder, some used an ellipsoid. The assumption of the bladder shape as a body of revolution is not necessary: for every symmetrically expanding body a relation between filling volume and change in wall thickness can be calculated rigorously.

Methods: The usual geometrical calculations are used in this procedure. The origin of the co-ordinate system is chosen in the lumen of the empty bladder. The smallest cross-sectional size is measured as $2x_0$ and defines the x-axis of the co-ordinate system. Perpendicular to this cross-section and mutually perpendicular the cross-sections along the y- and z-axes are measured $2y_0,2z_0$. The volume of the bladder V_t now is: $V_t=k\cdot x_0\cdot y_0\cdot z_0$. For a spherical or ellipsoid bladder $k=4\pi/3$, for a spherical bladder moreover $x_0=y_0=z_0=r$. For a bladder, degenerated to a rectangular parallelepiped (a box) k=8. With $y_0=a\cdot y_0$ and $z_0=b\cdot x_0$ $(a,b\geq 1)$, $V_t=k\cdot a\ b\cdot x_0^3=k^*\ x_0^3$. The wall thickness of the empty bladder in for instance the x direction is $\alpha\cdot x_0$ $(0\leq\alpha\leq 2)$; $\alpha=1$ when both "front" and "back" wall are equal-sized. When the bladder is filled with a volume V_i, it is assumed that the expansion of the bladder is symmetrical, that is linear in all three dimensions, meaning that after filling the total volume $V_e=V_i+V_t$ is calculated as $k\cdot x_e\cdot y_e\cdot z_e$. Because of the symmetry of the expansion, the bladder wall thickness also reduces linearly in the three dimensions or, stated otherwise, the inner shape is also symmetrical with the total shape, or equal to $k\cdot x_i\cdot y_i\cdot z_i$. Thus $V_e= k^*\cdot x_0^3$, $V_i= k^*\cdot x_0^3$. Linear expansion in the three dimensions means that $x_e/x_0=y_e/y_0=z_e/z_0=p_e$ and similar for p_i. The wall thickness of the filled bladder for instance in the direction x now is $d=x_e-x_i$, or $d=\alpha\cdot x_0\cdot(p_e-p_i)$. Cubing this equation and rewriting it as $d^3=\alpha^3\cdot p_i^3\cdot x_0^3\cdot(p_e/p_i-1)^3$, $d^3=\alpha^3\cdot V_i/K\cdot(\{p_e^3/p_i^3\}^{1/3}-1)^3$ $(K=k^*/\alpha^3)$, or $d^3=V_i/K\cdot(\{V_e/V_i\}^{1/3}-1)^3$ finally gives $d^3=(1/K)^{1/3}\cdot(V_e^{1/3}-V_i^{1/3})^3$. Because $V_e=V_i+V_t$ this means: $d=(V_t/K)^{1/3}\cdot(\{V_i/V_t+\}1^{1/3}-\ V_i/V_t)$. This relation holds generally for the change in wall thickness in any direction.

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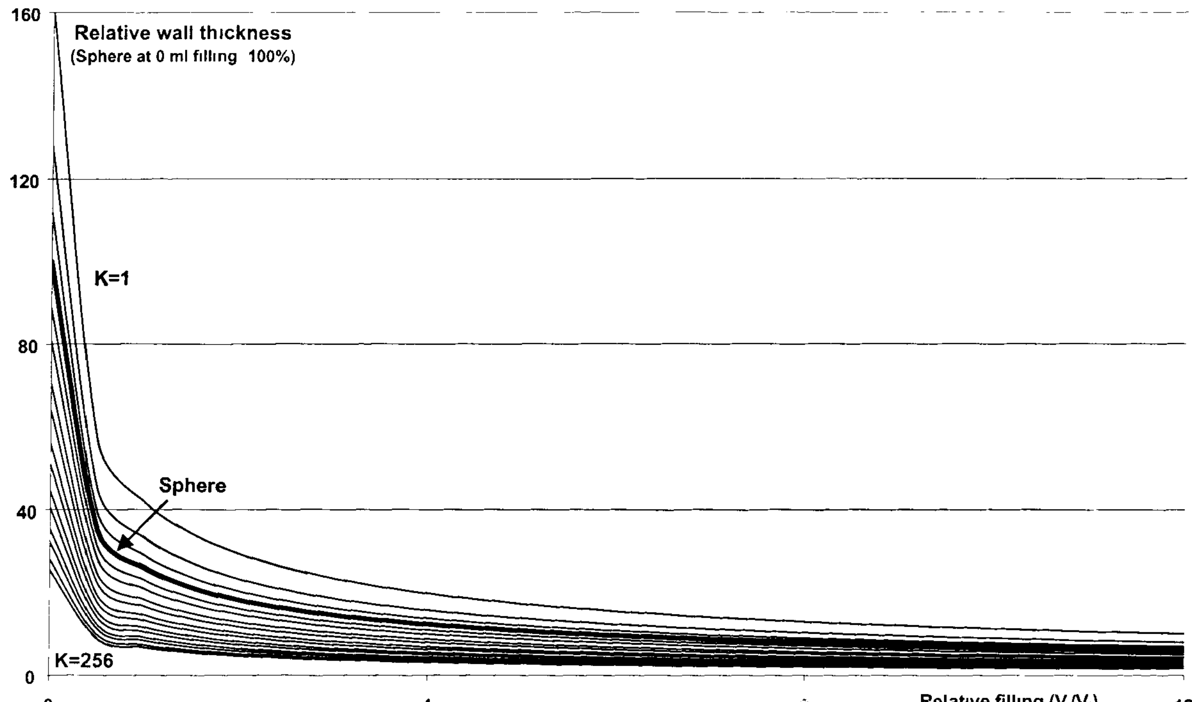
For each detrusor tissue volume a family of curves can now be created depicting the relation between d and the relative filling rate V_i/V_t. Reasonable assumptions for the eccentricities a and b to lie below 2 and for the wall thickness proportion α to be between 0.5 and 1.5 give a range for K between 1 and 256.

Results: A family of curves relating the bladder wall thickness and the filled volume relative to the tissue volume is presented for a bladder with a detrusor volume of 50 ml and a selection of geometry factors K between 1 and 256 (figure).



As is clear from the relation, all curves are parallel. The bumps in the figure are artefacts caused by the steps of $0.5V_t$ used in the present calculation. The curves above the spherical curve stem from using the larger section of the wall (α>1) in a bladder with unequal wall thickness.

Conclusion: The relation between bladder wall thickness and filling volume for any symmetrically expanding three-dimensional body is equal to that of the model of a spherical bladder, multiplied by a geometry factor K that incorporates the eccentricity of the bladder shape and its variation in wall thickness. This relation has been rigorously derived from the mathematical and geometrical equations.

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